Lecture „Item Response Theory“

Latent Class Analysis

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Basic Ideas of Latent Class Analysis (LCA)

- In the Latent Class Model, the latent variable, say ξ, has a *nominal scale*.

- Given a value of ξ, the category probabilities are constant, that is,

$$P(Y_i = y_i \mid U) = P(Y_i = y_i \mid \xi), \quad \forall\, y_i = 0, 1, \ldots, k_i, \quad i = 1, \ldots, m,$$

  where m is the number of items, $k_i + 1$ the number of values (i.e., response categories) of item Y_i.

- The category probabilities of an item are determined by the latent class.

Fictitious example - Learning styles of students

⇒ Question with response categories ‘agree’ and ‘disagree’

1. I learn using the material of the lecture and the exercise course.
2. I learn using the literature.
3. I learn in self-organized student groups.
4. I learn using material found in the internet.
5. I learn using the collection of examination questions.

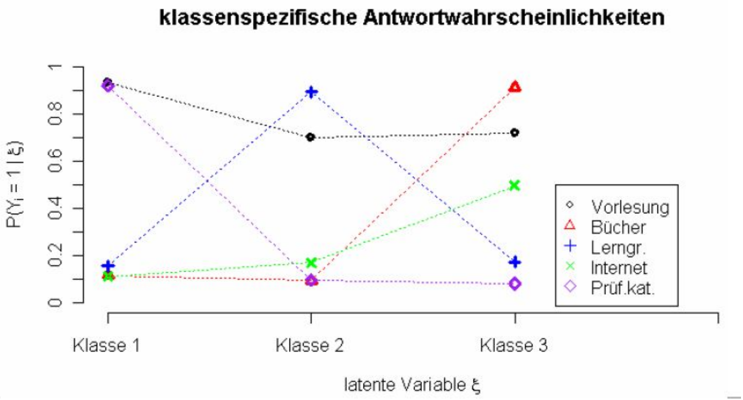
Fictitious example - Learning styles of students

- Student differ with respect to their *learning styles*.

LCA is the methodology for a **typology**. *LCA* is methodology for constructing a test aiming at measuring (latent) classes or types.

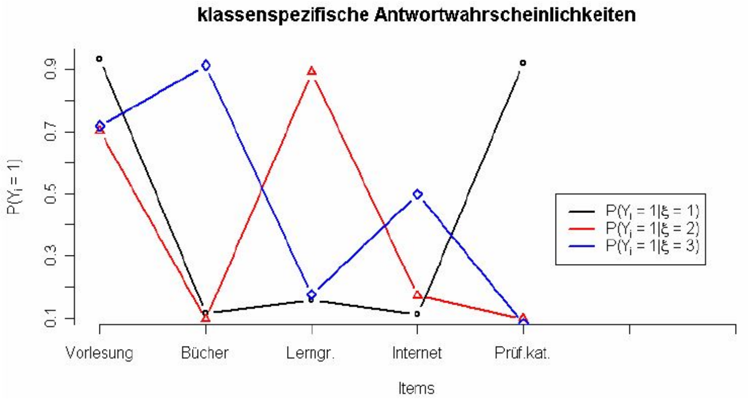
Fictitious example - Learning styles of students

- Response probabilities given the latent variable ξ with three values (the learning type).



Fictitious example - Learning styles of students

- An alternative presentation of the conditional response probabilities given ξ (**item profiles**)



Assumptions defining the latent class model

- $\xi = f(U)$ is a discrete (latent) variable with values $c = 1, \ldots, N_c$ and, according to the theorem of total probability,

$$P(Y_i{=}y_i) \;=\; \sum_{c=1}^{N_c} P(Y_i{=}y_i|\xi{=}c) \cdot P(\xi{=}c), \quad \forall\, i = 1, \ldots, m, \quad y_i = 0, 1, \ldots, k_i.$$

- ξ-conditional independence of the items Y_i, that is,

$$\forall\, i = 1, \ldots, m,\; y_i = 0, 1, \ldots, k_i,\; :$$
$$P(Y_i{=}y_i \mid U, Y_1, \ldots, Y_{i-1}, Y_{i+1}, ..., Y_m) = P(Y_i{=}y_i \mid \xi)$$

Implications of the assumptions

ξ-conditional independence of the items implies:

- ξ-conditional independence of the responses (i.e., of the events $\{Y_i = y_i\}$) from the person variable U

$$\forall\, i = 1, \ldots, m, \quad \forall\, y_i = 0, 1, \ldots, k_i : \quad P(Y_i = y_i \mid U) = P(Y_i = y_i \mid \xi).$$

  According to this equation, the conditional probability of a response $\{Y_i = y_i\}$ is only determined by ξ, and not by other attributes of the persons. (This is one of the causality conditions treated in the lecture of methods of evaluation research.)

- U-conditional probabilities of response patterns $y_1, \ldots, y_m$:

$$\forall\, (y_1, \ldots, y_m) \in \{1, \ldots, k_1\} \times \ldots \times \{1, \ldots, k_m\} :$$

$$P(Y_1 = y_1, \ldots, Y_m = y_m \mid U) = P(Y_1 = y_1, \ldots, Y_m = y_m \mid \xi)$$

$$= \prod_{i=1}^{m} P(Y_i = y_i \mid U) = \prod_{i=1}^{m} P(Y_i = y_i \mid \xi).$$

Model fit tests in LCA

- The null hypothesis that the true response pattern probabilities and the model implied response pattern probabilities are identical can be tested by a

1. Likelihood ratio test
2. Pearson χ^2-test
3. Cressie-Read test

A caveat:

If the number of expected frequencies of one of the response probabilities is smaller than five, then these tests might not be valid.

Bootstrap based model fit test is then still possible.