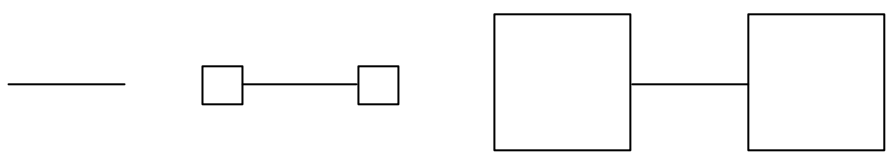
Contents

Example: Baldwin Illusion



(a) $X = 1$ (b) $X = 2$ (c) $X = 3$

Figure 12.1. Three Baldwin figures

Example: Baldwin Illusion

Table 12.1. Conditional distributions of the judgements.

Y (Judgement in mm)	$P(Y \mid X=1)$	$P(Y \mid X=2)$	$P(Y \mid X=3)$
13	0,1	0,0	0,1
14	0,2	0,0	0,2
15	0,4	0,05	0,4
16	0,2	0,1	0,2
17	0,1	0,2	0,1
18	0,0	0,3	0,0
19	0,0	0,2	0,0
20	0,0	0,1	0,0
21	0,0	0,05	0,0

Baldwin Illusion: Possible Values

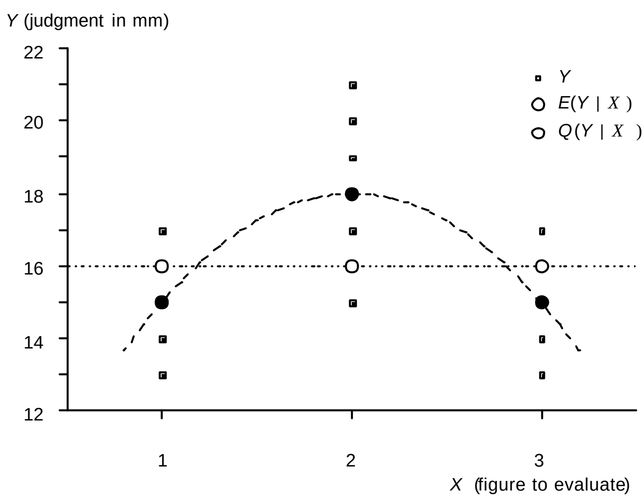


Figure 10.2. Idealized graph of the possible values of the judgments Y for the three figures to evaluate.

Baldwin Illusion: Conditional Variances

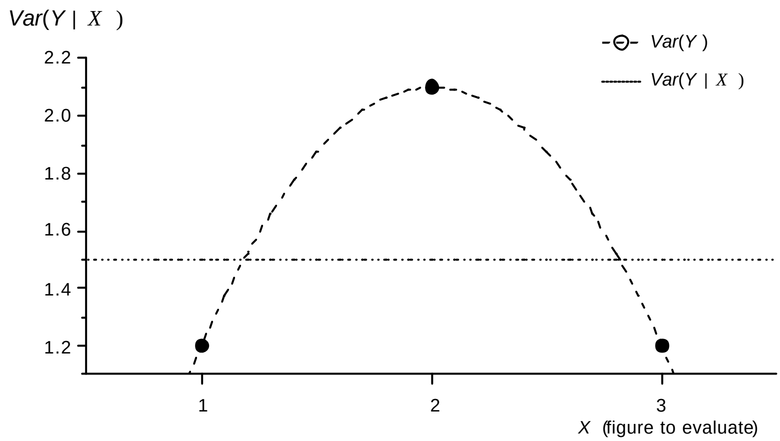


Figure 10.3. Graph of the conditional variance of the judgments variable Y for the three different values of X.

Conditional Covariance: Definition

Definition 12.1. Let Y_1 and Y_2 be two numerical random variables with finite expected values and finite variances and X a (single- or multidimensional) random variable (with any set of values), all on the same probability space. And let:

$$e_1 := Y_1 - E(Y_1 \mid X) \quad \text{and} \quad e_2 := Y_2 - E(Y_2 \mid X)$$

be the residuals of Y_1 and Y_2 respectively with respect to their regressions on X. The *conditional covariance* of Y_1 and Y_2 given X is then defined by:

$$Cov(Y_1, Y_2 \mid X) := E\big[[Y_1 - E(Y_1 \mid X)] \cdot [Y_2 - E(Y_2 \mid X)] \,\big|\, X\big]$$

$$= E(e_1 \cdot e_2 \mid X)$$

Conditional Variance: Definition

Definition 12.2. Let Y_1 and Y_2 be two numerical random variables with finite expected values and finite variances and X a (single- or multidimensional) random variable (with any set of values), all on the same probability space. The *conditional variance* of Y given X is the conditional covariance of Y with itself. In formulas:

$$Var(Y|X) := Cov(Y, Y|X).$$

The *conditional standard deviation of Y given X* is then defined by:

$$Std(Y|X) := +\sqrt{Var(Y|X)},$$

i.e. the positive square root of the conditional variance $Var(Y|X)$.

Conditional Variance: Rules of Computation

(i) $Var(Y|X) = E(Y^2|X) - E(Y|X)^2$

(ii) $Var(Y|X) = 0$, falls $Y = \alpha$

(iii) $Var(\alpha + Y|X) = Var(Y|X)$

(iv) $Var(\alpha \cdot Y|X) = \alpha^2\, Var(Y|X)$

(v) $Var(\alpha_1 Y_1 + \alpha_2 Y_2 | X)$

$$= \alpha_1^2\, Var(Y_1|X) + \alpha_2^2\, Var(Y_2|X) + 2\alpha_1\alpha_2 Cov(Y_1, Y_2|X)$$

(vi) $Var[f(X) \cdot Y|X] = f(X)^2\, Var(Y|X)$

(vii) $E[Var(Y|X)] = Var(e) = E(e^2)$

Conditional Covariance: Rules of Computation

(viii) $Cov(Y_1, Y_2 \mid X) = E(Y_1 \cdot Y_2 \mid X) - E(Y_1 \mid X) \cdot E(Y_2 \mid X)$

(ix) $Cov(Y_1, Y_2 \mid X) = 0$, falls $Y_1 = \alpha$ oder $Y_2 = \alpha$

(x) $Cov(\alpha_1 + Y_1, \alpha_2 + Y_2 \mid X) = Cov(Y_1, Y_2 \mid X)$

(xi) $Cov(\alpha_1 Y_1, \alpha_2 Y_2 \mid X) = \alpha_1 \alpha_2 Cov(Y_1, Y_2 \mid X)$

(xii) $Cov(\alpha_1 Y_1 + \alpha_2 Y_2, \beta_1 Z_1 + \beta_2 Z_2 \mid X)$

$$= \alpha_1 \beta_1 Cov(Y_1, Z_1 \mid X) + \alpha_1 \beta_2 Cov(Y_1, Z_2 \mid X)$$

$$+ \alpha_2 \beta_1 Cov(Y_2, Z_1 \mid X) + \alpha_2 \beta_2 Cov(Y_2, Z_2 \mid X)$$

(xiii) $Cov[f_1(X) \cdot Y_1, f_2(X) \cdot Y_2 \mid X] = f_1(X) \cdot f_2(X) \cdot Cov(Y_1, Y_2 \mid X)$.

(xiv) $E[Cov(Y_1, Y_2 \mid X)] = Cov(e_1, e_2) = E(e_1 \cdot e_2)$

Conditional Correlation: Definition

Definition 12.3. Given the assumptions of Definition 12.1. the *conditional correlation function* of two numerical random variables Y_1 and Y_2 given X is defined by:

$$Kor(Y_1, Y_2 \mid X) := \frac{Cov(Y_1, Y_2 \mid X)}{Std(Y_1 \mid X) \cdot Std(Y_2 \mid X)},$$

and the *conditional correlation* given $X = x$ is defined by:

$$Kor(Y_1, Y_2 \mid X = x) := \frac{Cov(Y_1, Y_2 \mid X = x)}{Std(Y_1 \mid X = x)\, Std(Y_2 \mid X = x)} \; .$$

Partial Correlation: Definition

Definition 12.3. Given the assumptions of Definition 12.1. the *partial correlation* of two numerical random variables Y_1 and Y_2 with respect to X is defined by:

$$Kor(Y_1, Y_2 . X) := \frac{Cov(e_1, e_2)}{Std(e_1) \cdot Std(e_2)},$$

where e_i , i = 1, 2, is the residual with respect to the regression $E(Y_i \mid X)$.

Partial Correlation: Theorem

Theorem 12.1. For the partial correlation of two numerical random variables Y_1 and Y_2 with respect to X the following equation is true:

$$Kor(Y_1, Y_2 . X) = \frac{Kor(Y_1, Y_2) - R_{Y_1|X} R_{Y_2|X} Kor[E(Y_1 \mid X), E(Y_2 \mid X)]}{\sqrt{1 - R^2_{Y_1|X}} \cdot \sqrt{1 - R^2_{Y_2|X}}}.$$

$R^2_{Y_1|X}$ und $R^2_{Y_2|X}$ denote the coefficients of determination of the two regressions $E(Y_i \mid X)$. If both of the regressions $E(Y_i \mid X) = \alpha_{i0} + \alpha_{i1} X$, i = 1, 2, are linear, the following equation holds as well:

$$Kor(Y_1, Y_2 . X) = \frac{Kor(Y_1, Y_2) - Kor(Y_1, X) \cdot Kor(Y_2, X)}{\sqrt{1 - Kor(Y_1, X)^2} \cdot \sqrt{1 - Kor(Y_2, X)^2}}.$$