Causal Regression: Paradoxes

What you are going to learn

- Simpson Paradox
- Individual causal effects and a Second Paradox
- The Role of Randomization
- The Role of Homogenous Populations

Simpson Paradox II

Table 15.1. Evaluation of a treatment

	Treatment		
Success	Yes (X = 1)	No (X = 0)	Total
Yes (Y = 1)	500	600	1100
No (Y = 0)	500	400	900
Total	1000	1000	2000

Note. From Novick (1980). The numbers are fictitious.

Regression Equation for one Regressor

Die lineare Regression

$$E(Y \mid X) = P(Y = 1 \mid X) = \alpha_0 + \alpha_1 X = 0.60 - 0.10 \cdot X$$

beschreibt dann die regressive Abhängigkeit in der Gesamtpopulation

Simpson Paradox I

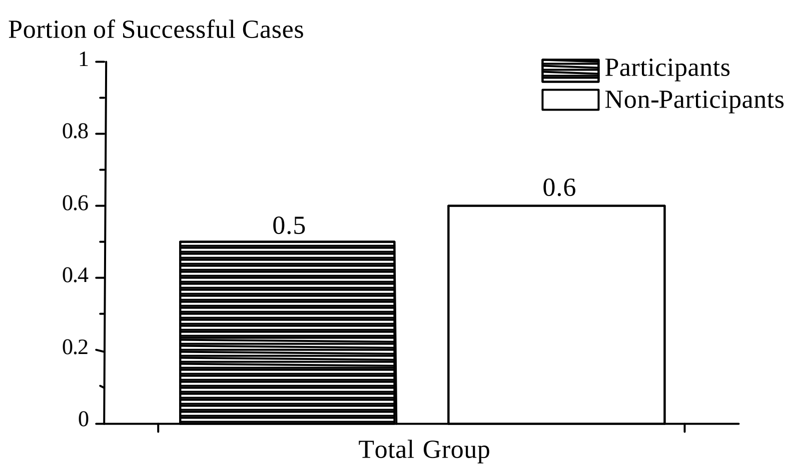

Figure 15.1. Histogram of the relative frequencies of successful cases.

Simpson Paradox IV

Table 15.2. Evaluation of a treatment separated for gender

A. Men ($W = 1$)

	Treatment		
Success	Yes ($X = 1$)	No ($X = 0$)	Total
Yes ($Y = 1$)	300	75	375
No ($Y = 0$)	450	175	625
Total	750	250	1000

B. Women ($W = 0$)

Success	Yes ($X = 1$)	No ($X = 0$)	Total
Yes ($Y = 1$)	200	525	725
No ($Y = 0$)	50	225	275
Total	250	750	1000

Note. From Novick (1980). The numbers are fictitious.

Regression Equation for two Regressors

$$E(Y \mid X, W) = P(Y = 1 \mid X, W) = \beta_0 + \beta_1 X + \beta_2 W$$

$$= 0.70 + 0.10 \cdot X - 0.40 \cdot W,$$

Simpson Paradox III

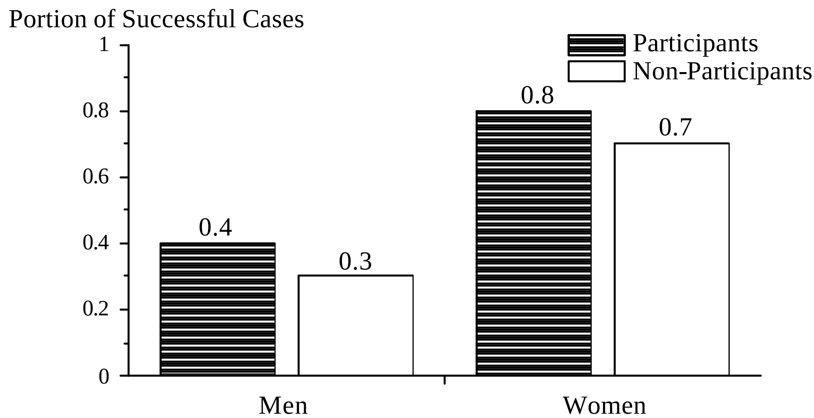

Figure 15.2. Histogram of the relative frequencies of successful cases, separated for men and women.

Example I: Comparable Groups

Table 15.3. Example to illustrate individual and average causal effects

	(a) Non-Comparable Groups			(b) Comparable Groups		
Unit	$\mu_{1,u}$	$\mu_{0,u}$	$\mu_{1,u} - \mu_{0,u}$	$\mu_{1,u}$	$\mu_{0,u}$	$\mu_{1,u} - \mu_{0,u}$
u_1	**82**	68	14	82	**68**	14
u_2	89	**81**	8	**89**	81	8
u_3	**101**	89	12	**101**	89	12
u_4	**108**	102	6	108	**102**	6
u_5	118	**112**	6	**118**	112	6
u_6	**131**	119	12	131	**119**	12
u_7	139	**131**	8	139	**131**	8
u_8	152	**138**	14	**152**	138	14
Average	**105.5**	**115.5**	10	**115**	**105**	10

Note. The bold values are the chosen values.

Example II: Randomization

Table 15.4. Example, in which there is a positive effect of the experimental group against the control group for every single person and the probability to be assigned to the experimental group is independent (pre last column) and dependent (last column) from the person.

Unit	$\mu_{1,u}$	$\mu_{2,u}$	$\mu_{1,u} - \mu_{2,u}$	$P(X = 1 \mid U = u)$	$P(X = 1 \mid U = u)$
u_1	82	68	14	1/2	8/9
u_2	89	81	8	1/2	7/9
u_3	101	89	12	1/2	6/9
u_4	108	102	6	1/2	5/9
u_5	118	112	6	1/2	4/9
u_6	131	119	12	1/2	3/9
u_7	139	131	8	1/2	2/9
u_8	152	138	14	1/2	1/9

Note. The probability $P(U = u)$ to choose one of the 8 persons is 1/8 and the (unconditional) probability $P(X = x)$, to be assigned to the experimental group, is 1/2.

Example III: Homogenous Population

Table 15.5. Example for a homogenous population

Unit	$\mu_{1,u}$	$\mu_{2,u}$	$\mu_{1,u} - \mu_{2,u}$
u_1	110	100	10
u_2	110	100	10
u_3	110	100	10
u_4	110	100	10
u_5	110	100	10
u_6	110	100	10
u_7	110	100	10
u_8	110	100	10
Average	**110**	**100**	**10**