Individual and Average Causal Effects

What you are going to learn

- The Underlying Random Experiment
- Basic Concepts
- Individual and Average Causal Effect
- Theorems

The Underlying Random Experiment I

$\Omega = \Omega_U \times \Omega_X \times \Omega_Y$ *set of possible outcomes*

Comprises the following components: “Draw a unit $u \in \Omega_U$, assign the unit (or observe its self-assignment) $\omega_X \in \Omega_X$ and register the value $\omega_Y \in \Omega_Y$. “

$U: \Omega \to \Omega_U$ *observational-unit variable*

Its value is the drawn unit („observational unit“) *u*.

$X: \Omega \to \Omega'_X$ *treatment variable*

Its value represents the (experimental) treatment condition, which the unit is assigned ω_X.

$Y: \Omega \to \mathrm{IR}$ *response variable*

The observed response variable, whose dependency from *X* should be causally interpreted.

The Underlying Random Experiment II

$E(Y \mid X)$ *treatment regression*

Describes the dependency of the response variable Y from the treatment variable X, which should be causally interpreted.

$E(Y \mid X, U)$ *unit-treatment regression*

Its values $\mu_{x,u} := E(Y \mid X = x,\ U = u)$ are the expected values of the response variable Y of a person (or a unit) u within a (treatment) condition $X = x$.

Individual and Average Causal Effect: Definition

Definition 16.1. Let X be a treatment variable, Y a response variable with positive and finite variance and U the observational-unit variable, all three on a common probability space $\langle\Omega, \mathfrak{A}, P\rangle$, where Ω has the structure given above. Further let $P(X = x, U = u) > 0$. The *individual causal effect* of x_1 vs. x_2 on (the expected value of) Y for the unit u is the difference

$$
\begin{aligned}
ICE_u(1, 2) &:= E(Y \mid X = x_1, U = u) - E(Y \mid X = x_2, U = u) \\
&= \mu_{1,u} - \mu_{2,u}.
\end{aligned}
$$

Average Causal Effect

Definition 16.2. Under the same assumptions as in Definition 16.1. the *average causal effect* of x_1 vs. x_2 on (the expected value of) Y is defined:

$$ACE(1, 2) := \sum_u ICE_u(1, 2)\, P(U = u).$$

The Prima Facie Effect

The *prima facie effect of* x_1 vs. x_2 on (the expected value of) Y is defined:

$$PFE(1, 2) := E(Y \mid X = x_1) - E(Y \mid X = x_2)$$

Causal Unbiasedness

Definition 16.3. Given the assumptions of Definition 16.1 the term

$$CUE(Y \mid X = x) := \sum_u \mu_{x,\, u}\, P(U = u)$$

is called the (*causally*) *unbiased expected value* of Y given $X = x$.

Definition 16.4. Given the assumptions of Definition 16.1 the regression $E(Y \mid X)$ and its values $E(Y \mid X = x)$ are called *causally unbiased*, if for every value x of X

$$E(Y \mid X = x) = CUE(Y \mid X = x).$$

Theorems

Theorem 16.1. (Randomization theorem) Given the assumptions of Definition 16.1. Stochastic independence of U and X implies causal unbiasedness of the regression $E(Y \mid X)$ and its values $E(Y \mid X = x)$, and:

$$PFE(1, 2) = ACE(1, 2).$$

Theorem 16.2. (Homogeneity theorem) Given the assumptions of Definition 16.1, conditional independence of the regressand Y of U given X, i.e.

$$E(Y \mid X, U) = E(Y \mid X),$$

implies causal unbiasedness of the regression $E(Y \mid X)$ and its values $E(Y \mid X = x)$, as well as $PFE_{12} = ACE_{12}$.