Basic Concepts 1

The set of possible outcomes of the random experiment

$$\Omega = \Omega_U \times \Omega_{S_1} \times \ldots \times \Omega_{S_t} \times \ldots \times \Omega_{S_n} \times \Omega_{O_1} \times \ldots \times \Omega_{O_t} \times \ldots \times \Omega_{O_n}$$

Test-score variables $Y_{it}: \Omega \rightarrow \mathrm{IR}$

Projections

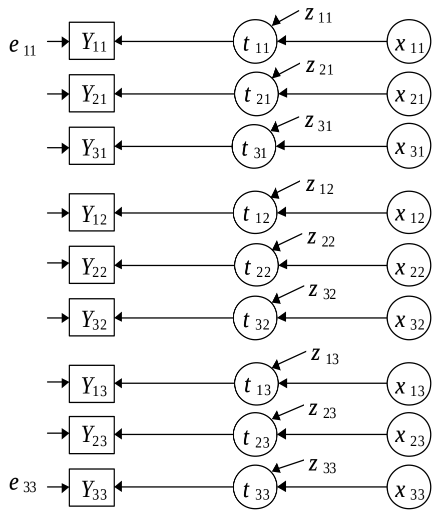
$U: \Omega \rightarrow \Omega_U$ person projection

$S_t: \Omega \rightarrow \Omega_{S_t}$ situation projection

Theoretical Variables 2

$t_{it} := E(Y_{it} \mid U, S_t)$	Latent state variable
$e_{it} := Y_{it} - t_{it}$	Measurement error variable
$x_{it} := E(Y_{it} \mid U)$	Latent trait variable
$z_{it} := t_{it} - x_{it}$	Latent state residual

Path Diagram



Properties of Latent Variables

Decomposition of variables

$Y_{it} = t_{it} + e_{it}$

$t_{it} = x_{it} + z_{it}$

Decomposition of variances

$Var(Y_{it}) = Var(t_{it}) + Var(e_{it})$

$Var(t_{it}) = Var(x_{it}) + Var(z_{it})$

Expected Values and Covariances of the Residuals

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$E(e_{it}) = 0$

$E(z_{it}) = 0$

$Cov(e_{it}, z_{js}) = 0$

$Cov(e_{it}, t_{js}) = 0$

$Cov(e_{it}, x_{js}) = 0$

$Cov(z_{it}, x_{js}) = 0$

Expected Values and Covariances of the Residuals

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Proof of: $E(z_{it}) = 0$

$E(z_{it}) = E(t_{it} - x_{it})$ (Inserting the definition)

$= E(t_{it}) - E(x_{it})$ (Rule iii of Rule Box 5.1)

$= E[E(Y_{it} \mid U, S_t)] - E[E(Y_{it} \mid U)]$ (Inserting the definitions)

$= E(Y_{it}) - E(Y_{it}) = 0$ (Rule iv of Rule Box 6.2)

Expected Values and Covariances of the Residuals

Proof of: $Cov(z_{it}, x_{js}) = 0$

First we proof $E(z_{it} | x_{js}) = 0$, which implies $Cov(z_{it}, x_{js}) = 0$.

$z_{it} = t_{it} - x_{it}$ (Definition of z_{it})

$= t_{it} - E(Y_{it} | U)$ (Definition of x_{it})

$= t_{it} - E[E(Y_{it} | U, S_t) | U]$ (Rule vi of Rule Box 6.2)

$= t_{it} - E(t_{it} | U)$ (Definition of t_{it})

This shows that z_{it} *is a residual* with respect to the regressor U. Hence, its regression on all functions of U is zero and $x_{js} := E(Y_{js} | U)$ is such a function of U. Furthermore, its covariances with all functions of U are zero, because the covariances of a residual with all functions of its regressor are zero.

Important Coefficients

Reliability

$$Rel(Y_{it}) = \frac{Var(t_{it})}{Var(Y_{it})} = Con(Y_{it}) + Spe(Y_{it})$$

Consistency

$$Con(Y_{it}) = \frac{Var(x_{it})}{Var(Y_{it})}$$

Occasion Specificity

$$Spe(Y_{it}) = \frac{Var(z_{it})}{Var(Y_{it})}$$

Stability of the latent state variable $Kor(t_{it}, t_{is})$

Stability of the latent trait variable $Kor(x_{it}, x_{is})$

Singletrait Model
Path Diagram

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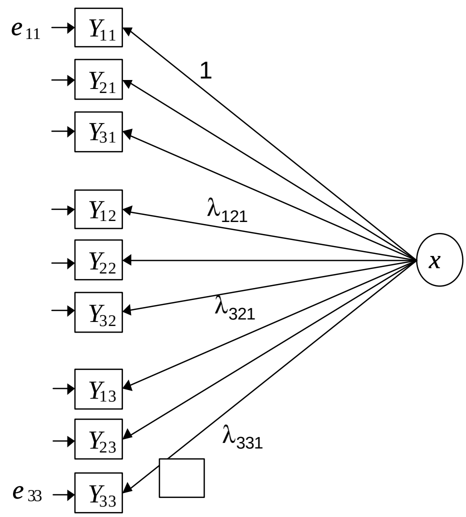

Singletrait Model
Definition

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$$Y_{it} = t_{it} + e_{it}$$

$$= \lambda_{it0} + \lambda_{it1}\, x + e_{it}$$

$$Cov(e_{it}, e_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Singletrait Model
Fixing the scale and Identification

Fixing the scale of the latent trait variable x:

$$E(x) = 0 \quad \text{and} \quad Var(x) = 1$$

or

$$\lambda_{110} = 0 \quad \text{and} \quad \lambda_{111} = 1$$

Identification in the case $\lambda_{110} = 0$ and $\lambda_{111} = 1$

$$E(x) = E(Y_{11})$$

$$\lambda_{it1}^2 Var(x) = \frac{Cov(Y_{it}, Y_{js})\ Cov(Y_{it}, Y_{ku})}{Cov(Y_{js}, Y_{ku})}, \quad (i, t) \neq (j, s) \neq (k, u)$$

$$Var(t_{it}) = \lambda_{it1}^2 Var(x)$$

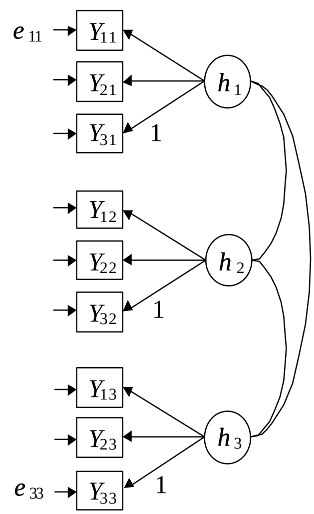
$$Con(Y_{it}) = Rel(Y_{it}) = \frac{\lambda_{it1}^2 Var(x)}{Var(Y_{it})}$$

Singletrait Model
Testability

$$\begin{aligned} Var(Y_{it}) &= Var(t_{it}) + Var(e_{it}) \\ &= \lambda_{it1}^2 Var(x) + Var(e_{it}) \end{aligned}$$

$$Cov(Y_{it}, Y_{js}) = Cov(t_{it}, t_{js}) = \lambda_{it1} \lambda_{js1} Var(x), \quad (i, t) \neq (j, s)$$

Multistate Model
path diagram



Multistate Model
Definition

$$Y_{it} = t_{it} + e_{it}$$

$$t_{it} = \lambda_{it0} + \lambda_{it1} h_t + e_{it}$$

$$Cov(e_{it}, e_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Multistate-Singletrait Model
path diagram

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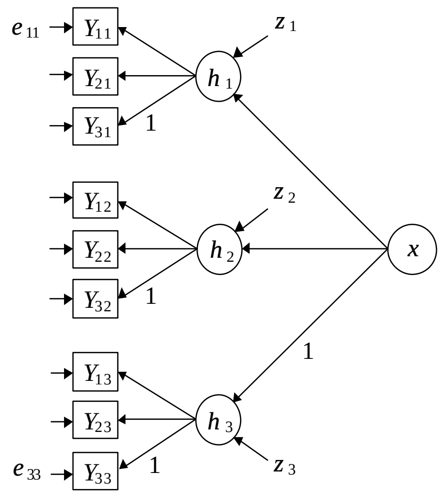

Multistate-Singletrait Model
Definition

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$$Y_{it} = t_{it} + e_{it}$$

$$= \lambda_{it0} + \lambda_{it1} h_t + e_{it}$$

$$h_t = \gamma_{t0} + \gamma_{t1} x + z_t$$

$$Cov(e_{it}, e_{js}) = 0 \qquad (i, t) \neq (j, s)$$

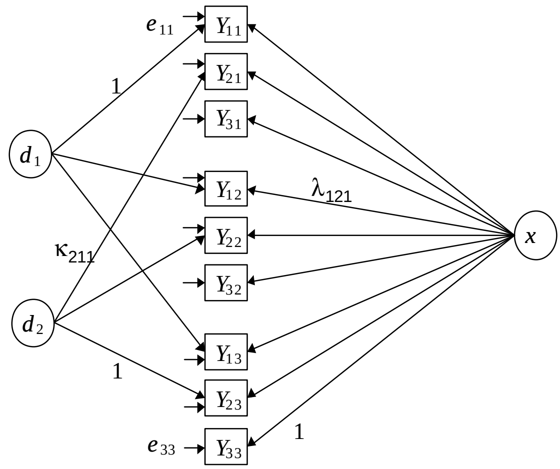
$$Cov(e_{it}, h_s) = 0$$

$$Cov(z_t, z_s) = 0$$

$$Cov(z_t, e_{js}) = 0$$

$$Cov(z_t, x) = 0$$

Singletrait Model with Method Factors
path diagram



Singletrait Model with Method Factor
Definition

$$Y_{it} = t_{it} + e_{it}$$
$$= \lambda_{it0} + \lambda_{it1} x + d_i + e_{it}$$
$$Y_{33} = t_{33} + e_{33}$$
$$= x + e_{33}$$

$$Cov(e_{it}, e_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Multistate Model with Method Factors

path diagram

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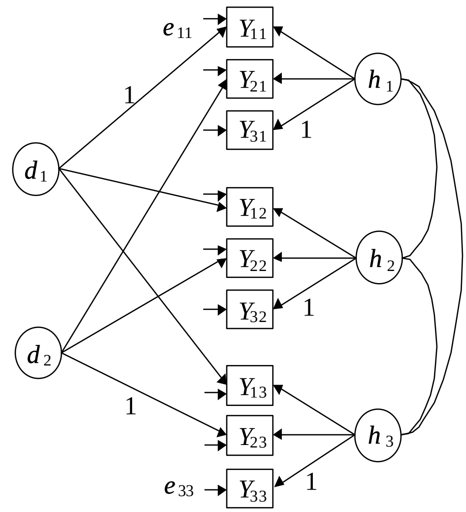

Singletrait-Multistate Model with Method Factors

path diagram

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