LST Modeling with Logistic Item Response Models

Purpose

Modeling the relationship between the item and time-specific latent variable and the manifest answer pattern $\boldsymbol{Y} = \boldsymbol{y}$

How?

For every item i we consider the relationship between the probability to answer in a particular category of this item ($Y_i = y$) and the item and time-specific latent variable.

Steyer, R. and Partchev, I. (2001). Latent state-trait modeling with logistic item response *models*. In: R. Cudeck, S. du Toit, and D. Sörbom (Eds.), *Structural Equation Modeling: Present and Future* (pp. 481-520). Chicago: Scientific Software International.

The Set of Possible Outcomes

$$\Omega = \Omega_U \times \Omega_{O_1} \times \ldots \times \Omega_{O_t} \times \ldots \times \Omega_{O_n}$$

$$\Omega = \Omega_U \times (\Omega_{S_1} \times \Omega_{O_1}) \times \ldots \times (\Omega_{S_t} \times \Omega_{O_t}) \times \ldots \times (\Omega_{S_n} \times \Omega_{O_n})$$

$$Y_{it}: \Omega \rightarrow \{0, 1, 2, \ldots, c_i\}$$

$$U: \Omega \rightarrow \Omega_U ,$$

$$S_t: \Omega \rightarrow \Omega_{S_t} ,$$

Assumptions

$$P(Y_{it} = 0 \mid U, S_t) = \frac{1}{1 + \sum_{j=1}^{c_{it}} \exp \sum_{k=1}^{j} (q_{it} + \mu_{it} - ?_{itk})}$$

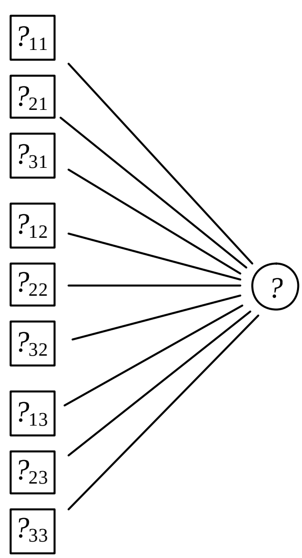
$$P(Y_{it} = y \mid U, S_t) = \frac{\exp \sum_{j=1}^{y} (q_{it} + \mu_{it} - ?_{itj})}{1 + \sum_{j=1}^{c_{it}} \exp \sum_{k=1}^{j} (q_{it} + \mu_{it} - ?_{itk})}, \quad y = 1, 2, \ldots, c_{it}$$

local stochastic independence

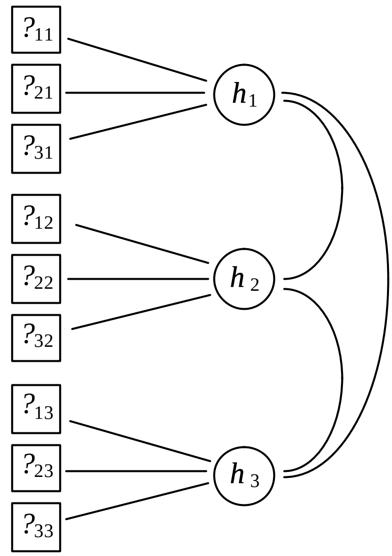
$P(Y_{it} = y \mid q_{it}, \boldsymbol{y}_{it}) = P(Y_{it} = y \mid q_{it})$,

where $\boldsymbol{y}_{it}$ denotes the vector of all other items Y_{js}, $(j, s) \neq (i, t)$.

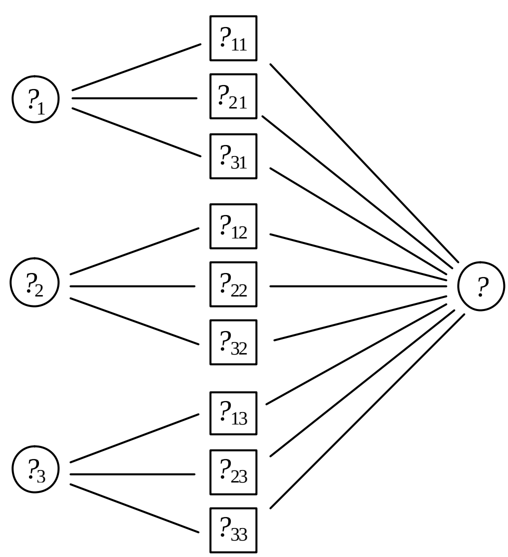
Singletrait Model:
path diagram



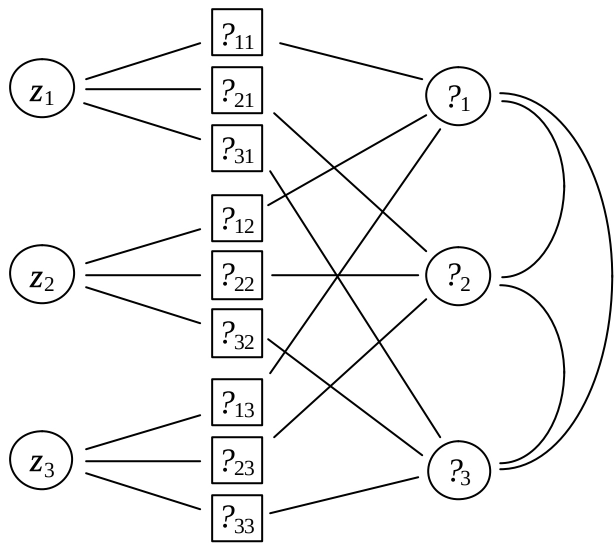
Multistate Model:
path diagram



Singletrait-Multistate Model:
path diagram



Multitrait-Multistate Model:
path diagram



Models for Latent Variables

Table 1. *Assumptions on the item-and-time-specific latent state variables* q_{it} *defining different latent state-trait models*

Model	Assumptions on qit	
(a) Singletrait	(a1) $q_{it} = x$ (a2) $E(x) = 0$	$i = 1, \ldots, m,$ $t = 1, \ldots, n$
(b) Multistate	(b1) $q_{it} = h_t$ (b2) $E(h_t) = 0$	$i = 1, \ldots, m,$ $t = 1, \ldots, n$

Models for Latent Variables

Model	Assumptions on q_{it}	
(c) Singletrait-Multistate	(c_1) $q_{it} = x + z_t$	$i = 1, \dots, m,$ $t = 1, \dots, n$
	(c_2) $E(x) = 0$	
	(c_3) $E(z_t) = 0$	$t = 1, \dots, n$
	(c_4) $Cov(x, z_t) = 0$	$t = 1, \dots, n$
	(c_5) $Cov(z_s, z_t) = 0$	s ¹ t $s, t = 1, \dots, n$
(d) Multitrait-Multistate	(d_1) $q_{it} = x_i + z_t$	$i = 1, \dots, m,$ $t = 1, \dots, n$
	(d_2) $E(x_i) = 0$	$i = 1, \dots, m$
	(d_3) $E(z_t) = 0$	$t = 1, \dots, n$
	(d_4) $Cov(x_i, z_t) = 0$	$i = 1, \dots, m,$ $t = 1, \dots, n$
	(d_5) $Cov(z_s, z_t) = 0$	s ¹ t $s, t = 1, \dots, n$

Models for the Item Parameters

In the partial credit model, we have

$$a := \sum_{i=1}^{m} \sum_{t=1}^{n} (c_{it} + 1)$$

response categories. Furthermore, there are at most

$$b_{max} := \sum_{i=1}^{m} \sum_{t=1}^{n} c_{it}$$

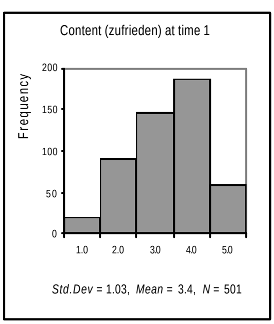
independent parameters (locations μ_{it} and thresholds κ_{itj}), where m and n denote the number of items and occasions, respectively.

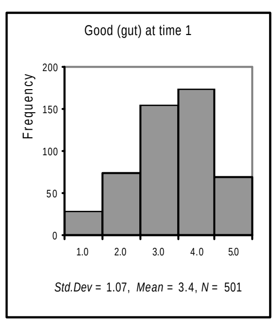
Models for Item Parameters

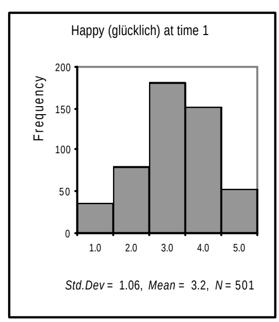
Table 2. Different models resulting from crossing five parameterizations of the locations parameters μ_{it} with three models for the threshold parameters κ_{itj}

		Models for the threshold parameters κ_{itj}			
		Equality of threshold distances within items			
		yes		no	
		Equality of thresholds across items			
		yes		no	
		Invariance of thresholds over time			
		yes	no	yes	no
Models for the location parameters μ_{it}	μ	P1111	P1112	P1121	P1122
	$\mu + \alpha_i$	P2111	P2112	P2121	P2122
	$\mu + \beta_t$	P3111	P3112	P3121	P3122
	$\mu + \alpha_i + \beta_t$	P4111	P4112	P4121	P4122
	$\mu + \alpha_i + \beta_t + \gamma_{it}$	P5111	P5112	P5121	P5122

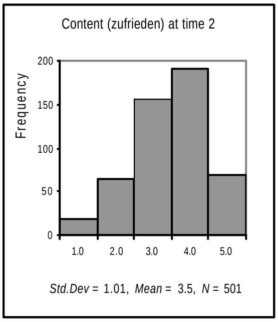
Diagrams –Items at Time 1

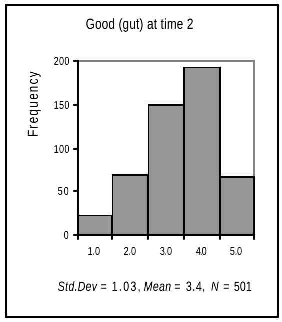


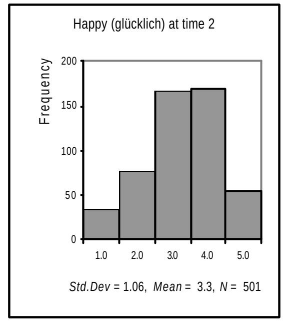




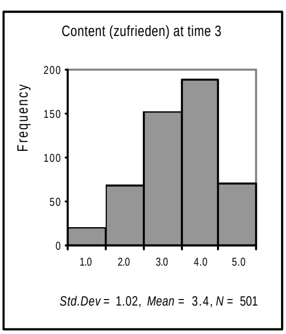
Diagrams –Items at Time 2

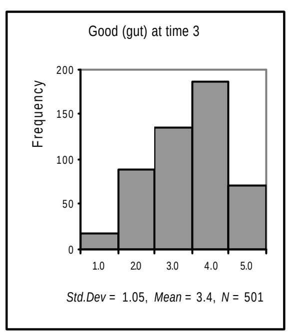


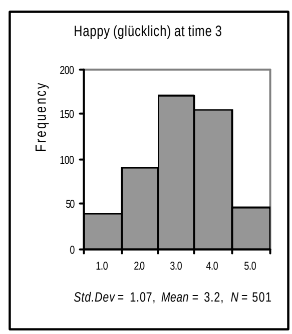




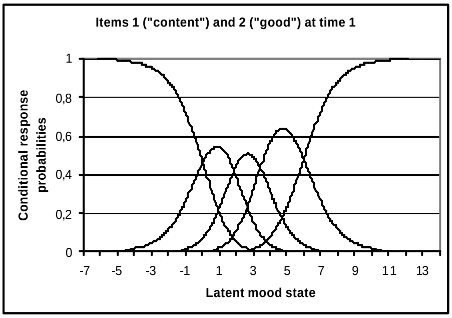
Diagrams –Items at Time 3

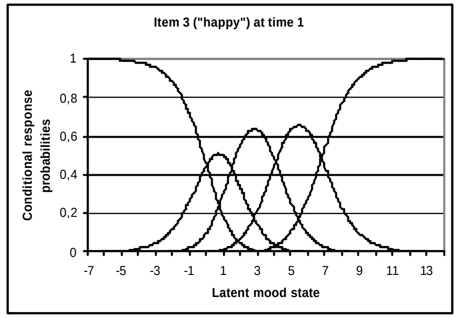




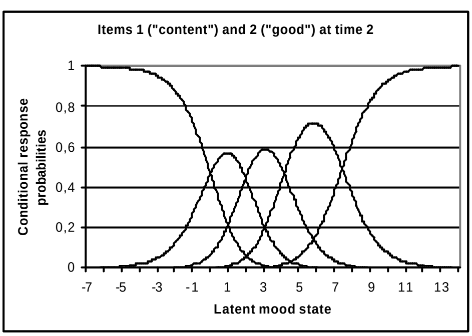


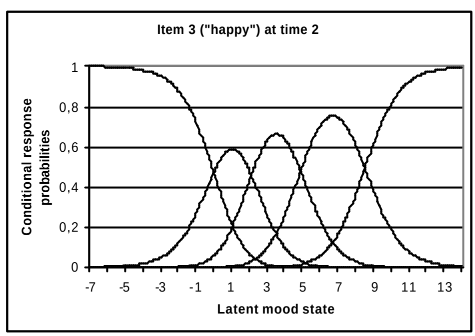
Conditional Response Probabilities
– Items at Time 1 -



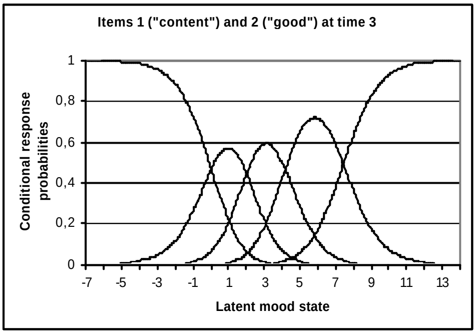


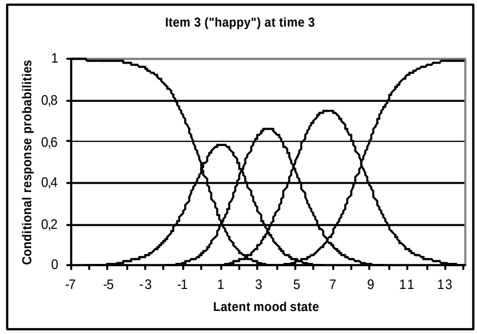
Conditional Response Probabilities
– Items at Time 2 -





Conditional Response Probabilities
– Items at Time 3 -





Item Parameter Estimates I

Table 5. *Item parameter estimates with standard errors and fit statistics*

			Unweigthed Fit		Weigthed Fit	
Parameter	Estimate	SE	MNSQ	T	MNSQ	T
μ	3.476	0.02	1.06	0.92	1.03	0.48
β_2	0.961	0.04	1.11	1.74	1.13	1.98
β_3	0.838	0.04	1.13	1.97	1.15	2.20
$\kappa_{112} = \kappa_{212}$	1.848	0.10	0.94	-0.92	1.09	1.12
$\kappa_{113} = \kappa_{213}$	3.396	0.08	0.97	-0.51	1.05	0.76
$\kappa_{114} = \kappa_{214}$	5.967	0.12	0.81	-3.20	1.06	0.61
κ_{312}	1.489	0.15	0.67	-6.00	0.97	-0.36
κ_{313}	4.076	0.13	0.80	-3.31	1.01	0.19
κ_{314}	6.780	0.19	0.97	-0.49	1.22	1.67

Deviance and Number of Parameters

Table 3. *Models with their deviance and number of parameters*

Model	Deviance	Convergence	Number of parameters in the model for the		
			Latent variables	Expectations	Thresholds
0	11013.34	no	45	9	27
A	11017.07	no	9	9	27
B	10993.67	yes	6	9	27
C	11089.00	yes	4	9	27
D	13233.31	yes	6	9	1
E	11037.84	yes	6	9	9
F	11027.24	yes	6	9	9
G	11005.71	yes	6	9	12
H	11034.98	yes	6	1	12
I	11005.20	yes	6	7	12
J	11024.54	yes	6	7	12
K	11013.13	yes	6	5	12
L	11017.64	yes	6	3	12

Covariances and Correlations

Table 4. *Covariances and correlations of the traits and state residuals*

	z_1	z_2	z_3	x_1	x_2
z_1	2.161 (0.137)	0	0	0	0
z_2	0	4.088 (0.258)	0	0	0
z_3	0	0	4.631 (0.293)	0	0
x_1	0	0	0	0.872 (0.055)	.814
x_2	0	0	0	1.301 (0.101)	2.928 (0.185)

Item Parameter Estimates II

$\kappa_{122} = \kappa_{222}$	1.979	0.08	1.36	5.19	1.23	2.71
$\kappa_{123} = \kappa_{223}$	4.146	0.06	1.02	0.35	1.07	1.01
$\kappa_{124} = \kappa_{224}$	7.425	0.09	0.89	-1.74	1.17	1.93
κ_{332}	2.118	0.11	1.07	1.10	0.98	-0.20
κ_{333}	4.889	0.10	0.87	-2.09	0.95	-0.71
κ_{334}	8.513	0.14	0.60	-7.42	1.30	2.60

Note: The standard errors are computed with the quick option described in Wu et al. (1998, p. 137). These approximations are computed ignoring the covariances of the parameter estimates.

Expectations, Variances, Consistency

Table 6. *Expectations, variances, consistency, and occasion specificity coefficients of the item-and-time-specific latent variables*

	Expectations	Variances	Consistency	Occasion Specificity
q_{11}	3.476	3.033	0.288	0.712
q_{21}	3.476	3.033	0.288	0.712
q_{31}	3.476	5.089	0.575	0.425
q_{12}	4.437	4.960	0.176	0.824
q_{22}	4.437	4.960	0.176	0.824
q_{32}	4.437	7.015	0.417	0.583
q_{13}	4.314	5.504	0.159	0.841
q_{23}	4.314	5.504	0.159	0.841
q_{33}	4.314	7.559	0.387	0.613

Estimated, Centered, Standardized Thresholds I

Table 7. *Estimated, centered, and standardized thresholds for all items at each occasion of measurement*

κ_{111}	0.000	-3.476	-1.996	
κ_{112}	1.848	-1.628	-0.935	
κ_{113}	3.396	-0.080	-0.046	
κ_{114}	5.967	2.491	1.430	-0.387
κ_{211}	0.000	-3.476	-1.996	
κ_{212}	1.848	-1.628	-0.935	
κ_{213}	3.396	-0.080	-0.046	
κ_{214}	5.967	2.491	1.430	-0.387
κ_{311}	0.000	-3.476	-1.541	
κ_{312}	1.489	-1.987	-0.881	
κ_{313}	4.076	0.600	0.266	
κ_{314}	6.780	3.304	1.465	-0.173

Estimated, Centered, Standardized Thresholds II

κ_{121}	0.000	-4.437	-1.992	
κ_{122}	1.979	-2.458	-1.104	
κ_{123}	4.146	-0.291	-0.131	
κ_{124}	7.425	2.988	1.342	-0.471
κ_{221}	0.000	-4.437	-1.992	
κ_{222}	1.979	-2.458	-1.104	
κ_{223}	4.146	-0.291	-0.131	
κ_{224}	7.425	2.988	1.342	-0.471
κ_{321}	0.000	-4.437	-1.675	
κ_{322}	2.118	-2.319	-0.876	
κ_{323}	4.889	0.452	0.171	
κ_{324}	8.513	4.076	1.539	-0.210

Estimated, Centered, Standardized Thresholds III

κ_{131}	0.000	-4.314	-1.839	
κ_{132}	1.979	-2.335	-0.995	
κ_{133}	4.146	-0.168	-0.072	
κ_{134}	7.425	3.111	1.326	-0.395
κ_{231}	0.000	-4.314	-1.839	
κ_{232}	1.979	-2.335	-0.995	
κ_{233}	4.146	-0.168	-0.072	
κ_{234}	7.425	3.111	1.326	-0.395
κ_{331}	0.000	-4.314	-1.569	
κ_{332}	2.118	-2.196	-0.799	
κ_{333}	4.889	0.575	0.209	
κ_{134}	8.513	4.199	1.527	-0.158