Partial-Credit-Model

Purpose

Modeling the relationship between the person trait $x_u := f(U = u)$ to be measured and the manifest response pattern $\boldsymbol{Y} = \boldsymbol{y}$

How?

For every item *i* we consider the dependence of the probability to answer in a particular category of this item ($Y_i = y$) on the trait score ξ_u. This trait score of the person *u* determines the response probabilities in all items considered.

Person-Conditional Probabilities

person-conditional probabilities of categories

$$P(Y_i = y \mid U = u)$$

$$P(Y_i = y \mid x = \xi_u)$$

expressed as *regression:*

$$P(Y_i = y \mid U) := E(I_{Y_i = y} \mid U)$$

The Set of Possible Outcomes

The set of possible outcomes of the random experiment considered is:

$$\Omega = \Omega_U \times \Omega_O$$

where:

$$\Omega_O = \Omega_{O1} \times \Omega_{O2} \times \ldots \times \Omega_{Om}$$

m: number of items in the questionnaire and

$$\Omega_{Oi} = \{0, 1, \ldots, c_i\}$$

the set of response categories of item *i*.

Definition of the Random Variables

Definition of the (multidimensional) random variable *Y*

$$\boldsymbol{Y} = (Y_1, \ldots, Y_i, \ldots, Y_m)'$$

with

$$Y_i : \Omega \rightarrow \{0, 1, 2, \ldots, c_i\} \subset \mathbb{N}_0$$

The values for $\boldsymbol{Y} = (Y_1, \ldots, Y_i, \ldots, Y_m)'$ are:

$$\boldsymbol{y} = (y_1, \ldots, y_i, \ldots, y_m)',$$

where $y_i \in \{0, 1, 2, \ldots, c_i\}$ and c_i is the number of categories of item *i*.

U (person-variable)

$U: \Omega \rightarrow \Omega_U$ with $U(\omega) = u$.

Threshold Probabilities

$PT_{Uiy}: \Omega \to [0, 1]$

$PT_{Uiy}(\omega): = PT_{uiy}$

with: $PT_{Uiy} := P\left[Y_i = y \mid U, \left(\{Y_i = y-1\} \cup \{Y_i = y\}\right)\right]$

for every $y \in \{1, ..., c_i\}$

$$PT_{Uiy} = \frac{P(Y_i = y \mid U)}{P(Y_i = y-1 \mid U) + P(Y_i = y \mid U)}$$

Assumptions of the Partial Credit Model

(a) Rasch Homogeneity

$$PT_{uiy} = \frac{\exp(\xi_u - \kappa_{ij})}{1 + \exp(\xi_u - \kappa_{ij})} \qquad j = 1, \ldots, c_i$$

where

ξ_u : is the person parameter

κ_{ij} : is the item- and category specific threshold parameter

(b) local stochastic independence

$P(Y_i = y \mid x, \boldsymbol{y}_i) = P(Y_i = y \mid x)$

where

$\boldsymbol{y}_i = (Y_1, \ldots, Y_{i-1}, Y_{i+1}, \ldots, Y_m)$.

Category Probabilities

Consequences following from the assumptions

$$P(Y_i = 0 \mid U) = \frac{1}{1 + \sum_{j=1}^{c_i} \exp \sum_{k=1}^{j} (x - \lambda_{ik})}$$

and

$$P(Y_i = y \mid U) = \frac{\exp \sum_{j=1}^{y} (x - \lambda_{ij})}{1 + \sum_{j=1}^{c_i} \exp \sum_{k=1}^{j} (x - \lambda_{ik})},$$

with $y = 1, 2, \ldots, c_i$ (see Rost, 1996, for a derivation).

Item Parameters and Threshold Parameters

The item difficulty parameters κ_i in the Rasch Model for dichotomous items are replaced by the *threshold parameters* λ_{ij} in the Partial Credit Model, where the first index again refers to item i, and the second index refers to the first, the second etc. threshold (and pair of neighbored categories).

If the scores of each item Y_i are 0, 1, 2, …, c_i, i.e., $y \in \{0, 1, 2, \ldots, c_i\}$, we have c_i thresholds λ_{ij} for each item.